Ex-11 A train is timmed to run from Howrah to Delhi at an average speed of u kilometres per hour. Due to some engine trouble the starting of the train was delayed by l hours. At Delhi the speed is raised to v kilometres per hour in order that the train may reach in time. If the train runs with this increased speed up to a station at a distance s Kilometres from ~~bet~~ Delhi, then show that $s(v-u) = uvh$.

Soln Let the distance between Howrah and Delhi be d Kilometre.

Therefore first $(d-s)$ ~~dis~~ Kilometres, the train runs with speed u Km/hr and last s Kilometre with speed v Km/hr.

The scheduled time taken by the train from Howrah to Delhi ~~be~~ is $\frac{d}{u}$.

Due to some engine trouble, the time taken by the train is $\frac{d-s}{u} + \frac{s}{v}$.

By the condition

$$\frac{d-s}{u} + \frac{s}{v} + h = \frac{d}{u}$$

Or $\frac{d}{u} - \frac{s}{u} + \frac{s}{v} + h = \frac{d}{u}$

or $\frac{s}{u} - \frac{s}{v} = h$ or $s(v-u) = huv$

or $s(v-u) = uvh$ (proved)

Ex-12 Find the relative velocity of two railway trains running with a speed of 30 miles per hour and 50 feet per second respectively, when they are travelling i) in the same direction, ii) in the opposite directions.

Soln Let us first assume that the both the trains move in a straight line. Then their speed and velocity are ~~equivalent~~ same.

The ~~ve~~ velocity of first train $= v_1 = 30$ miles/hr

$$= \frac{30 \times 1760 \times 3}{60 \times 60} \text{ feet/sec}$$

$= 44$ feet/sec

Velocity of the second train $= v_2 = 50$ feet/sec.

(i) When both trains moves in the same direction, the relative velocity of second train with respect to the first train $= v_2 - v_1 = 6$ feet/sec

The relative velocity of first train with respect to the second train is also 6 feet/sec but in the opposite direction.

(ii) When the two train moves in the opposite directions, the relative velocity of second train with respect to the first train $= (50+44)$ feet/sec

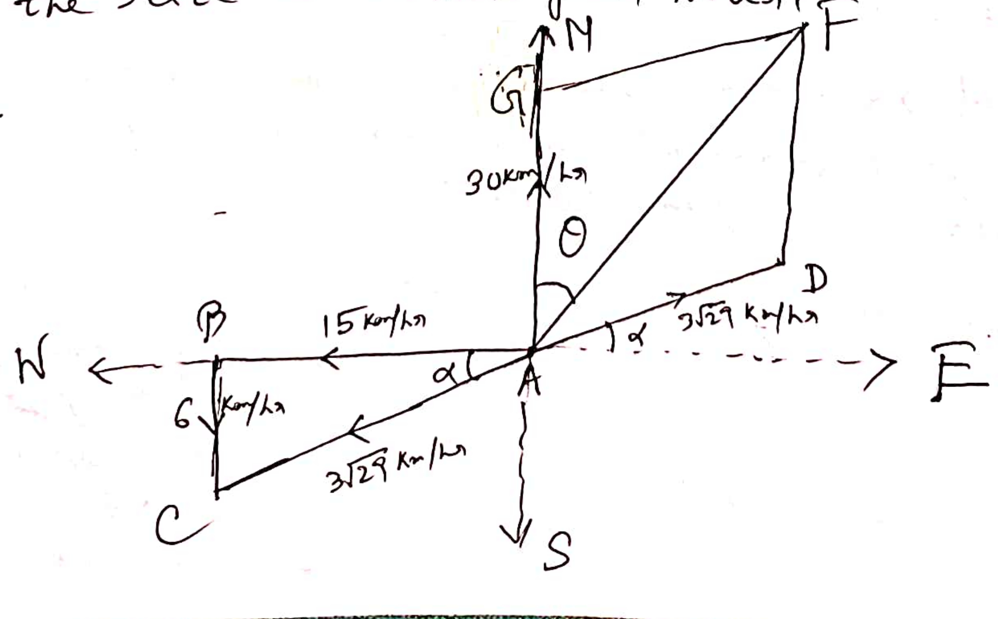
ie 94 feet/sec.

In this case the relative velocity of first train with respect to the second train is also 94 feet/sec but in the opposite direction.

Ex-13

A ship steams due west at the rate of 15 kms per hour when the river is flowing at the rate 6 kms per hour due south. What is the velocity relative to the ship of a train going due north at the rate of 30 kms per hour?

Soln



The ship has the velocity 15 km/hr in the West direction and the river is flowing with velocity 6 km/hr towards south. We representy these two velocity in the figure as $\vec{AB}$ and $\vec{BC}$ respectively. We first find the resultant of these two velocities. The magnitude of the resultant of these two velocities is $\sqrt{15^2+6^2} = \sqrt{261}$
$= 3\sqrt{29}$

Let the resultant velocity makes an angle α with west direction. Then $\tan\alpha = \frac{6}{15} = \frac{2}{5}$ --- (1)

A train is going towards north with a velocity 30 km/hr. Now we have to find the relative velocity of the train with respect to the ship. To get this we first reverse the resultant velocity of ship as $\vec{AD}$, then we have to find the resultant velocity of the train and the reversed velocity as mentioned above. Let the resultant velocity make an angle θ with the north direction

Then $\tan\theta = \dfrac{3\sqrt{29}\sin(90°-\alpha)}{30+3\sqrt{29}\cos(90°-\alpha)}$ [Since the angle between AD and AG is $90°-\alpha$]

$$= \frac{3\sqrt{29}\cos\alpha}{30+3\sqrt{29}\sin\alpha}$$

$$= \frac{3\sqrt{29}\cdot\frac{5}{\sqrt{29}}}{30+3\sqrt{29}\cdot\frac{2}{\sqrt{29}}} = \frac{15}{36} = \frac{5}{12}$$

$\therefore \theta = \tan^{-1}\left(\frac{5}{12}\right)$

Therefore the required relative velocity makes an angle makes an angle $\tan^{-1}\left(\frac{5}{12}\right)$ east of ~~with the~~ north. ~~direction~~

Magnitude of the relative ~~vel~~ velocity

$$= \sqrt{(30)^2 + (3\sqrt{29})^2 + 2 \times 30 \times 3\sqrt{29} \cos(90^\circ - \alpha)}$$

[$\because$ Angle between AD and AG is $90-\alpha$)]

$$= \sqrt{900 + 261 + 2 \times 30 \times 3 \times \sqrt{29} \sin\alpha}$$

$$= \sqrt{1161 + 2 \times 30 \times 3 \times \sqrt{29} \times \frac{2}{\sqrt{29}}}$$

$$= \sqrt{1161 + 360} = \sqrt{1521} = 39$$

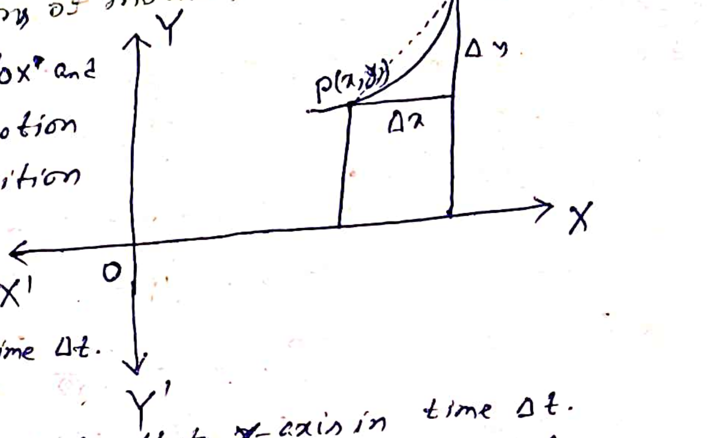
Hence the required relative velocity is 39 Km/hr at an angle $\tan^{-1}\left(\frac{5}{12}\right)$ east of north.

Motion in two dimensions

Expressions for velocity and accel in Cartesian Co-ords

~~(Q) 1) Find the expressions in terms of derivatives~~

(Q) 1) A particle moves on a plane. Find the expressions for the velocity components and acceleration components in cartesian co-ordinates with the equations of motion.



A set of rectangular axes X'OX' and Y'OY' are taken in the plane of motion of the particle. P(x, y) is the position of the particle at time t.

$Q(x+\Delta x, y+\Delta y)$ be the position of the particle after a small time Δt.

$\therefore \Delta x =$ displacement of the particle // to x-axis in time Δt.

$\Delta y =$ " " " // " y-axis " " Δt

$v_x =$ components of vel. // to x-axis at P.

$=$ rate of change of displacement // to x-axis.

$$= \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} = \dot{x}$$

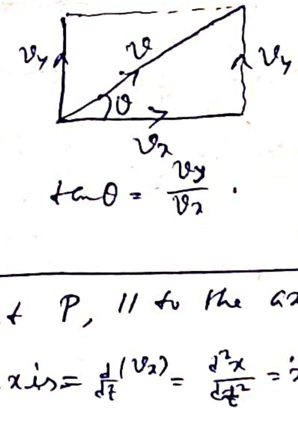
Similarly, $v_y =$ Component of vel. // to y-axis at P $= \frac{dy}{dt} = \dot{y}$

Resultant vel. $v = \sqrt{v_x^2 + v_y^2} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

If the velocity ~~to~~ v makes an angle θ with the x-axis then

$$\tan\theta = \frac{v_y}{v_x} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{dy}{dx}$$

= slope of the tangent at P(x, y) to the path of the particle. ~~at~~ ~~pt~~



$\tan\theta = \frac{v_y}{v_x}$.

This proves that the direction of ~~motion~~ vel. is always along the tangent at that pt to the path.

Let f_x and f_y be the components of acceleration at P, // to the axes.

$\therefore f_x =$ rate of change of velocity at P // to x-axis $= \frac{d}{dt}(v_x) = \frac{d^2x}{dt^2} = \ddot{x}$

Again $f_x = \frac{dv_x}{dt} = \frac{dv_x}{dx} \cdot \frac{dx}{dt} = \frac{dv_x}{dx} \cdot v_x = v_x \frac{dv_x}{dx}$.

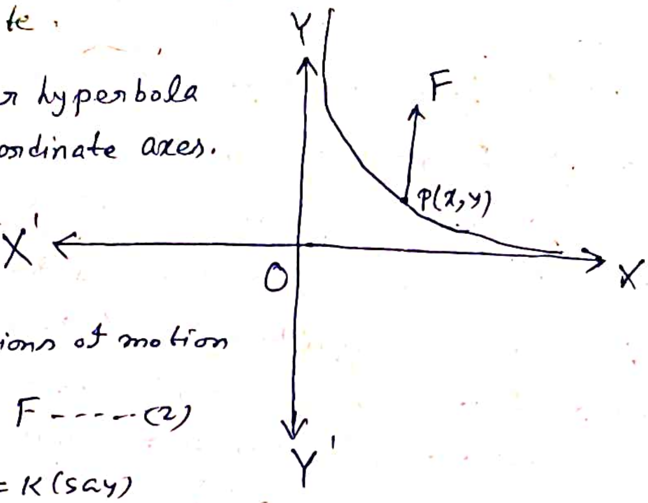
$$\therefore \quad f_x = \frac{d^2x}{dt^2} \text{ or } \frac{dv_x}{dt} \text{ or } v_x \frac{dv_x}{dx}.$$

Similarly $$f_y = \frac{d^2y}{dt^2} \text{ or } \frac{dv_y}{dt} \text{ or } v_y \frac{dv_y}{dy}$$

If m be the mass of the ~~velocity~~ particle, X and Y be the components of the force acting on the particle, // to the axes, then the equations of motion are,

$$m\frac{d^2x}{dt^2} = X, \quad m\frac{d^2y}{dt^2} = Y.$$

Ex-1 A particle describes a rectangular hyperbola under a force which is always ∥ to an asymptote, prove that the force varies as the cube of the distance from the other asymptote.



Let the equation of the rectangular hyperbola be $xy = c$, whose asymptotes are the co-ordinate axes.

Let $P(x,y)$ be the position of the particle at time t. F be the force ∥ to y-axis (one asymptote).

m = mass of the particle. The equations of motion are,

$$m\frac{d^2x}{dt^2} = 0 \quad ---(1) \qquad m\frac{d^2y}{dt^2} = F \quad ----(2)$$

from (1) $\frac{d^2x}{dt^2} = 0$, or, $\frac{dx}{dt} = \text{const} = K \text{ (say)}$

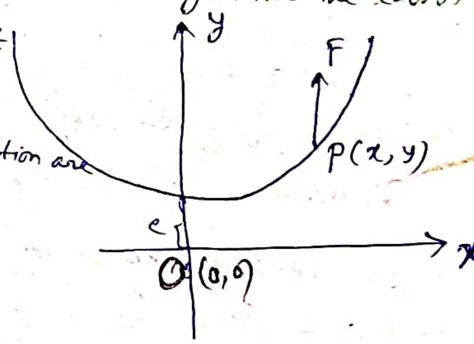
$xy = c$, or, $y = c/x$. or, $\frac{dy}{dt} = -\frac{c}{x^2}\cdot\frac{dx}{dt} = -\frac{Kc}{x^2}$

$$\frac{d^2y}{dt^2} = \frac{2Kc}{x^3}\cdot\frac{dx}{dt} = \frac{2cK^2}{x^3}$$

from (2) $m.\frac{2cK^2}{x^3} = F$ or, $F = \frac{2mcK^2}{(c/y)^3} = \frac{2mK^2}{c^2}y^3$

$\therefore$ $F \propto y^3$, where y is the distance of P from the other asymptote (x-axis)

Ex-2 A particle describes a catenary $y = c\cosh(x/c)$ under a force which is always ∥ to the axis of the catenary. Find the law of force.



Let F be the force on the particle at $P(x,y)$ at time t, ∥ to y-axis.

m = mass of the particle. The equations of motion are

$$m\frac{d^2x}{dt^2} = 0 \quad ---(1)$$

$$m\frac{d^2y}{dt^2} = F \quad ---(2)$$

from (1) $\frac{d^2x}{dt^2} = 0$, $\therefore \frac{dx}{dt} = \text{constant} = K \text{ (say)}$

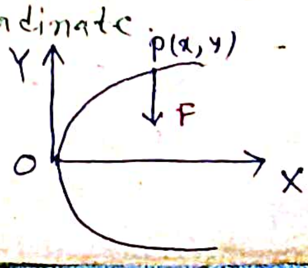
$y = c\cosh(x/c)$ $\therefore \frac{dy}{dt} = \sinh(x/c)\frac{dx}{dt} = K\sinh(x/c)$

$$\frac{d^2y}{dt^2} = \frac{K}{c}\cosh(x/c)\frac{dx}{dt} = \frac{K^2}{c}\cosh(x/c) = \frac{K^2}{c}\cdot\frac{y}{c} = \frac{K^2}{c^2}y$$

from (2) $m\frac{K^2}{c^2}y = F$, or $F \propto y$.

This is the law of force.

Ex-3 A particle describes a parabola under a force which is always directed perpendicularly towards its axis. Prove that the force is inversely proportional to the cube of the ordinate.



Let the equation of the parabola be

$$y^2 = 4ax.$$

The equations of motion are, $m\frac{d^2x}{dt^2}=0$ ---(1), $m\frac{d^2y}{dt^2}=-F$ --- (2)

From (1) $\frac{d^2x}{dt^2}=0$ or $\frac{dx}{dt}=$ constant $=K$ (say)

$y^2=4ax$, $2y\frac{dy}{dt}=4a\frac{dx}{dt}$ or, $\frac{dy}{dt}=\frac{2aK}{y}$

or, $\frac{d^2y}{dt^2}=-\frac{2aK}{y^2}\dot{y}$.

$\therefore$ From (2) $F=m.\frac{2aK}{y^2}.\frac{2aK}{y}=\frac{4a^2K^2m}{y^3}$, $\therefore F\propto\frac{1}{y^3}$ (proved)

Ex-4 A particle moves on a plane in such a way that its vel. components $\parallel$ to the axes at any instant are $u+wy$ and $v+w'x$ respectively, where u, v, w, w' are constants. Show that the path traced out by the particle is a conic section.

Let $P(x,y)$ be the position of the particle at time t.

By the condition, $\frac{dx}{dt}=u+wy$ ---(1) $\frac{dy}{dt}=v+w'x$ ---(2)

(1) $\div$ (2) gives $\frac{\frac{dx}{dt}}{\frac{dy}{dt}}=\frac{u+wy}{v+w'x}=\frac{dx}{dy}$, or, $dx(v+w'x)=(u+wy)dy$

Integrating, $vx+w'\frac{x^2}{2}=uy+\frac{wy^2}{2}+c/2$

or, $2vx+w'x^2=2uy+wy^2+c$

or $w'x^2-wy^2+2vx-2uy-c=0$ ---(3)

From the initial conditions c can be known, then (3) is the equation of the path of the particle.

Since the equation is of 2nd degree in x,y, it represents a conic section.

Ex-5. The curve $x=a(\theta-e\sin\theta)$, $y=a(1-e\cos\theta)$, where a,e are consts, and θ is a parameter, is described by a particle under the action of a force $\parallel$ to the x-axis. Show that the force varies as, $\frac{e-\cos\theta}{\sin^3\theta}$.

CH-05

Let $P(x,y)$ be the position of the particle at time t. F be the force $\parallel$ to x-axis, m = mass of the particle. The equations of motion are,

$m\frac{d^2x}{dt^2}=F$ ----(1) $m\frac{d^2y}{dt^2}=0$ ---- (2)

From (2) $\frac{d^2y}{dt^2}=0$, $\therefore\frac{dy}{dt}=$ const $=K$ (say) or, $\frac{d}{dt}\{a(1-e\cos\theta)\}=K$

or, $ae\sin\theta.\frac{d\theta}{dt}=K$ or, $\frac{d\theta}{dt}=\frac{K}{ae\sin\theta}$,

$\frac{d^2\theta}{dt^2}=-\frac{K}{ae}\operatorname{cosec}\theta\cot\theta.\dot{\theta}=-\frac{K}{ae}.\operatorname{cosec}\theta\cot\theta.\frac{K}{ae\sin\theta}=-\frac{K^2}{a^2e^2}\operatorname{cosec}^2\theta.\cot\theta$

$x=a(\theta-e\sin\theta)$ $\therefore\frac{dx}{dt}=a(\dot{\theta}-e\cos\theta.\dot{\theta})=a(1-e\cos\theta).\dot{\theta}$

or, $\frac{dx}{dt}=y\dot{\theta}$, $\frac{d^2x}{dt^2}=\dot{y}\dot{\theta}+y.\ddot{\theta}=K.\frac{K}{ae\sin\theta}+a(1-e\cos\theta).(-\frac{K^2}{a^2e^2}\operatorname{cosec}^2\theta\cot\theta)$

$=\frac{K^2}{ae\sin\theta}-\frac{K^2}{ae^2}(\operatorname{cosec}^2\theta\cot\theta-e\cos\theta\operatorname{cosec}^2\theta\cot\theta)$

$=\frac{K^2}{ae\sin\theta}\left[1-\frac{1}{e}.\frac{\cos\theta}{\sin^2\theta}(1-e\cos\theta)\right]$

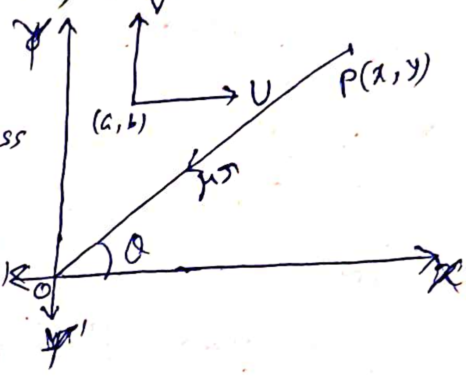
$=\frac{K^2}{ae\sin\theta}.\frac{e\sin^2\theta-\cos\theta+e\cos^2\theta}{e\sin^2\theta}=\frac{K^2}{ae\sin\theta}.\frac{e-\cos\theta}{e\sin^2\theta}=\frac{K^2}{ae^2}.\frac{e-\cos\theta}{\sin^3\theta}$.

From (1) $\therefore F=m.\frac{d^2x}{dt^2}=\frac{mK^2}{ae^2}.\frac{e-\cos\theta}{\sin^3\theta}$, $\therefore F\propto\frac{e-\cos\theta}{\sin^3\theta}$ (proved).

Ex. 1 A particle is moving in a plane under the action of an attracting force to a fixed pt. in the plane, equal to μ times the distance from that pt per unit mass. The initial co-ordinates and vel. components with respect to fixed rectangular axes passing through the centre of force ~~are~~ are (a,b) and (U,V) respectively. Find the co-ordinates (x,y) of the particle at time t and show that the path of the particle is given by,

$$\mu(bx-ay)^2+(Uy-Vx)^2=(bU-aV)^2.$$

90



Let $P(x,y)$ be the position of the particle at time t. $OP=r$.

The force on the particle is μr per unit mass towards O. $\angle POX=\theta$.

The equations of motion are,

$$\frac{d^2x}{dt^2}=-\mu r\cos\theta=-\mu r.\frac{x}{r}=-\mu x \text{ --- (1)}$$

$$\frac{d^2y}{dt^2}=-\mu r\sin\theta=-\mu y \text{ --- (2)}$$

From (1) $\frac{d^2x}{dt^2}+\mu x=0$.

The general solⁿ of this equation is $x=C_1\cos\sqrt{\mu}t+C_2\sin\sqrt{\mu}t$.

Similarly " " " " the equation (2) " $y=C_3\cos\sqrt{\mu}t+C_4\sin\sqrt{\mu}t$.

$$\frac{dx}{dt}=-C_1(\sin\sqrt{\mu}t)\sqrt{\mu}+C_2\sqrt{\mu}\cos\sqrt{\mu}t$$

$$\frac{dy}{dt}=-C_3\sqrt{\mu}\sin\sqrt{\mu}t+C_4\sqrt{\mu}\cos\sqrt{\mu}t$$

~~Wt~~ When $t=0$, $x=a$, $y=b$. $\therefore a=C_1$, $b=C_3$.

When $t=0$, $\frac{dx}{dt}=U$, $\frac{dy}{dt}=V$ $\therefore U=C_2\sqrt{\mu}$, $V=C_4\sqrt{\mu}$.

$$\therefore x=a\cos\sqrt{\mu}t+\frac{U}{\sqrt{\mu}}\sin\sqrt{\mu}t \text{ --- (3)}$$

$$y=b\cos\sqrt{\mu}t+\frac{V}{\sqrt{\mu}}\sin\sqrt{\mu}t \text{ --- (4)}$$

(3) and (4) give the co-ordinates (x,y) of the particle at time t.

Eliminating t from (3) and (4) we get the equation of the path.

~~Solving (3) and (4)~~.

(3) $\times V$ − (4) $\times U$ gives, $xV-yU=\cos\sqrt{\mu}t\,(aV-bU)$

or $\cos\sqrt{\mu}t=\frac{xV-yU}{aV-bU}$ ---- (5)

(3) $\times b$ − (4) $\times a$ gives, $xb-ya=\frac{1}{\sqrt{\mu}}\sin\sqrt{\mu}t\,(bU-Va)$

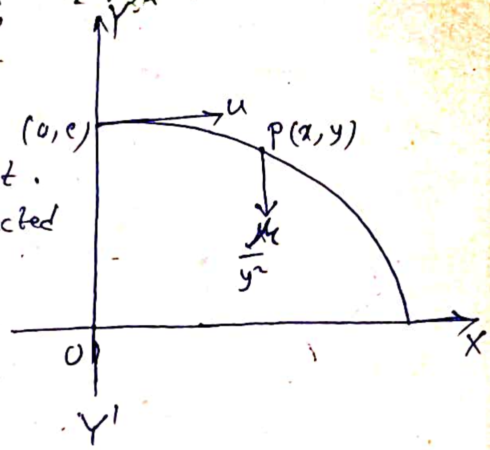
or, $\sin\sqrt{\mu}t=\frac{\sqrt{\mu}(xb-ya)}{(bU-Va)}$ --- (6)

$(5)^2+(6)^2$ gives. $1=\frac{(xV-yU)^2}{(aV-bU)^2}+\frac{\mu(xb-ya)^2}{(aV-bU)^2}$

or, $\mu(bx-ay)^2+(Uy-Vx)^2=(bU-aV)^2$ (proved)

Ex-7 A particle moves in a plane being attracted by a force perpendicular to a fixed st. line in it, equal to $\mu \div$ (distance from the line)^2 per unit mass. When at a distance c from the line, it is projected with a velocity u // to the line. Show that the particle strikes the line after a time $\frac{\pi}{2}\sqrt{\frac{c^3}{2\mu}}$.

x-axis is taken along the fixed line and y-axis is taken along the perpendicular to the line, through the pt of projection. The pt of projection is $(0, c)$.



Let $P(x, y)$ be the position of the particle at time t.

$\therefore$ The force per unit mass is $\frac{\mu}{y^2}$, // to y-axis and directed to the x-axis.

The equations of motion are,

$$\frac{d^2x}{dt^2} = 0 \quad \text{---(1)}, \qquad \frac{d^2y}{dt^2} = -\frac{\mu}{y^2} \quad \text{---(2)}$$

from (1) $\frac{dx}{dt} = \text{const} = C_1$. At $(0, c)$, $\frac{dx}{dt} = u$

$\therefore C_1 = u$, $\therefore \frac{dx}{dt} = u$.

from (2), multiplying both sides by $2\frac{dy}{dt}$ and then integrating,

$$\left(\frac{dy}{dt}\right)^2 = \frac{2\mu}{y} + C_2 \quad . \quad \text{At } (0, c),\ y = c,\ \frac{dy}{dt} = 0.$$

$$\therefore 0 = \frac{2\mu}{c} + C_2 \qquad \therefore C_2 = -\frac{2\mu}{c}.$$

$$\therefore \left(\frac{dy}{dt}\right)^2 = 2\mu\left(\frac{c-y}{cy}\right), \qquad \therefore \frac{dy}{dt} = -\sqrt{\frac{2\mu}{c}}\sqrt{\frac{c-y}{y}}$$

$\left[\because y \text{ decreases with } t, \therefore \frac{dy}{dt} < 0\right]$

or, $$-\frac{\sqrt{y}}{\sqrt{c-y}}\,dy = \sqrt{\frac{2\mu}{c}}\,dt.$$

Let the particle strikes the x-axis after time t_1.

when $t = 0$, $y = c$

$t = t_1$, $y = 0$.

$$\text{or} \quad -\int_c^0 \frac{\sqrt{y}\,dy}{\sqrt{c-y}} = \sqrt{\frac{2\mu}{c}}\int_0^{t_1} dt$$

$$\text{or} \quad \sqrt{\frac{2\mu}{c}}\,t_1 = \int_0^c \frac{\sqrt{y}\,dy}{\sqrt{c-y}}.$$

Let $y = c\sin^2\theta$

$\therefore dy = 2c\sin\theta\cos\theta\,d\theta$

when $y = 0$, $\theta = 0$

" $y = c$, $\theta = \frac{\pi}{2}$

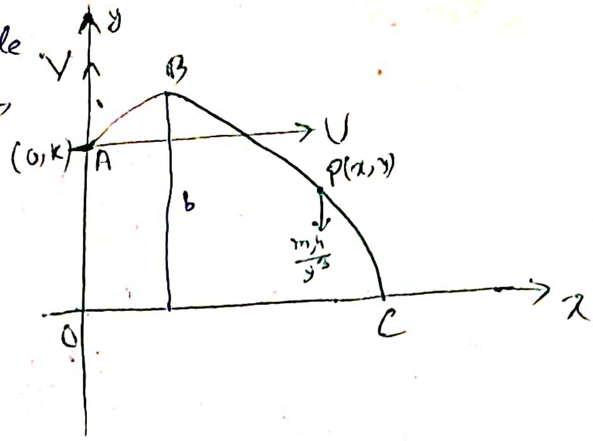
$$\therefore \sqrt{\frac{2\mu}{c}}\,t_1 = \int_0^{\pi/2} \frac{\sqrt{c}\sin\theta \cdot 2c\sin\theta\cos\theta\,d\theta}{\sqrt{c}\cos\theta}$$

$$= c\int_0^{\pi/2}(1 - \cos 2\theta)\,d\theta = c\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi/2} = c\,\frac{\pi}{2}$$

$$\therefore t_1 = \frac{\pi}{2}\sqrt{\frac{c^3}{2\mu}} = \text{required time}.$$

Ex-8 A particle of mass m is moving under the influence of an attractive
CH-09 force $\frac{m\mu}{y^3}$ towards the x-axis. Show that, if it be projected from the point $(0, k)$ with ~~component velocities~~ velocity components U and V // to the axes, It will not strike the axis of x unless $\mu > V^2k^2$ and that in this case the distance of the pt of impact from the origin is $\frac{Uk^2}{\sqrt{\mu} - Vk}$

Let $P(x,y)$ be the position of the particle at time t. The equations of motion are,



$$m\frac{d^2x}{dt^2} = 0 \quad \ldots (i)$$

$$m\frac{d^2y}{dt^2} = -\frac{m\mu}{y^3} \quad \ldots\ldots (ii)$$

From (i) integrating $\frac{dx}{dt} = C_1$.

at $(0,K)$ $\frac{dx}{dt} = U$, $\therefore C_1 = U$

$\therefore \frac{dx}{dt} = U$ or $dx = U\,dt$

Integrating, $x = Ut + C_2$

at $t=0$, $x=0$, $\therefore C_2 = 0$.

$\therefore x = Ut \quad \ldots\ldots (iii)$

From (ii) $\frac{d^2y}{dt^2} = -\frac{\mu}{y^3}$

multiplying both sides by $2\frac{dy}{dt}$ and integrating, $\left(\frac{dy}{dt}\right)^2 = \frac{\mu}{y^2} + C_3$

At $t=0$, $y=K$, $\frac{dy}{dt} = V$

$\therefore V^2 = \frac{\mu}{K^2} + C_3 \qquad \therefore C_3 = V^2 - \frac{\mu}{K^2}$

$\therefore \left(\frac{dy}{dt}\right)^2 = \frac{\mu}{y^2} + V^2 - \frac{\mu}{K^2}$

~~From (iv) and~~ or, $\left(\frac{dy}{dt}\right)^2 = \frac{\mu}{y^2} - \frac{\mu}{K^2} + V^2 = \frac{\mu}{y^2} + \frac{V^2K^2 - \mu}{K^2} \quad \ldots (iv)$

If $\mu < V^2K^2$, the R.H.S is always positive $\forall$ values of y.

$\therefore \left(\frac{dy}{dt}\right)^2 > 0$ always. $\therefore$ The particle will never strike the x-axis, since $\frac{dy}{dt}$ will never be zero and changes its sign.

$\therefore$ For striking x-axis we must have $\mu > V^2K^2$.

$\therefore$ The above equation can be written as,

$$\left(\frac{dy}{dt}\right)^2 = \frac{\mu}{y^2} - \frac{\mu - V^2K^2}{K^2} \quad \ldots (v)$$

Let $\frac{dy}{dt} = 0$, when $y = b$.

$\therefore 0 = \frac{\mu}{b^2} - \frac{\mu - V^2K^2}{K^2}$

$\therefore b^2 = \frac{\mu K^2}{\mu - V^2K^2} \Rightarrow \frac{\mu - V^2K^2}{K^2} = \frac{\mu}{b^2} \quad \ldots (vi)$

$\therefore \left(\frac{dy}{dt}\right)^2 = \frac{\mu}{y^2} - \frac{\mu}{b^2} \quad \therefore \frac{dy}{dt} = \sqrt{\mu}\sqrt{\frac{1}{y^2} - \frac{1}{b^2}} = \frac{\sqrt{\mu}}{yb}\sqrt{b^2 - y^2}$.

From A to B $\frac{dy}{dt} > 0$.

or, $y\frac{dy}{\sqrt{b^2-y^2}} = \frac{\sqrt{\mu}}{b}dt$, $\quad \int_K^b \frac{y\,dy}{\sqrt{b^2-y^2}} = \frac{\sqrt{\mu}}{b}\int_0^{t_1} dt$ [t_1 = time from A to B]

Let $b^2 - y^2 = z^2$ when $y = K$, $z = \sqrt{b^2 - K^2}$

$\therefore -2y\,dy = 2z\,dz$ $\qquad y = b$, $z = 0$.

$$\therefore \int_{\sqrt{b^2-K^2}}^{0} \frac{-z\,dz}{z} = \frac{\sqrt{\mu}}{b}(t_1-0)$$

or $\left[-z\right]_{\sqrt{b^2-K^2}}^{0} = \frac{\sqrt{\mu}}{b} t_1$, or $\sqrt{b^2-K^2} = \frac{\sqrt{\mu}}{b} t_1$ or $\frac{b\sqrt{b^2-K^2}}{\sqrt{\mu}} = t_1$

Let the particle strikes the x-axis at C.

From B to C y decreases with t.

$\therefore \frac{dy}{dt} < 0$, ie $\frac{dy}{dt} = -\frac{\sqrt{\mu}}{yb}\sqrt{b^2-y^2}$

ie $-\int_{b}^{0} \frac{y\,dy}{\sqrt{b^2-y^2}} = \frac{\sqrt{\mu}}{b}\int_{0}^{t_2} dt$ [t_2 = time from B to C]

or $\int_{0}^{b} \frac{y\,dy}{\sqrt{b^2-y^2}} = \frac{\sqrt{\mu}}{b}\int_{0}^{t_2} dt$ $\therefore t_2 = \frac{b^2}{\sqrt{\mu}}$.

$\therefore$ Total time ~~fr~~ from A to C, is $t_1+t_2 = \frac{b(b+\sqrt{b^2-K^2})}{\sqrt{\mu}}$

~~ie total time is~~

$$= \frac{1}{\sqrt{\mu}}\left[\frac{\mu K^2}{\mu - V^2K^2} + \frac{\sqrt{\mu}\,K}{\sqrt{\mu - V^2K^2}}\cdot\sqrt{\frac{\mu K^2}{\mu - V^2K^2} - K^2}\right] \text{[using (vi)]}$$

$$= \frac{1}{\sqrt{\mu}}\left[\frac{\mu K^2}{\mu - V^2K^2} + \frac{\sqrt{\mu}K \cdot K^2 V}{\mu - V^2K^2}\right] = \frac{K^2(\sqrt{\mu}+VK)}{(\sqrt{\mu}+VK)(\sqrt{\mu}-VK)}$$

$$= \frac{K^2}{\sqrt{\mu}-VK}.$$

ie. total time is $\frac{K^2}{\sqrt{\mu}-VK}$, From ~~x~~(iii) ~~Ut=x~~ $Ut = x$.

ie $x = U(t_1+t_2)$ $\therefore \overline{OC} = U\cdot\frac{K^2}{\sqrt{\mu}-VK} = \frac{UK^2}{\sqrt{\mu}-VK}$.